Embedding Hindsight Reasoning in Separation Logic

Roland Meyer, Thomas Wies, Sebastian Wolff

[PLDI'23]

Motivation

"*Programs = Algorithms + Data Structures*"

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Verification
via Separation Logic (SL)

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Automation

Example: `contains(5)`

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sorted({3,7}):
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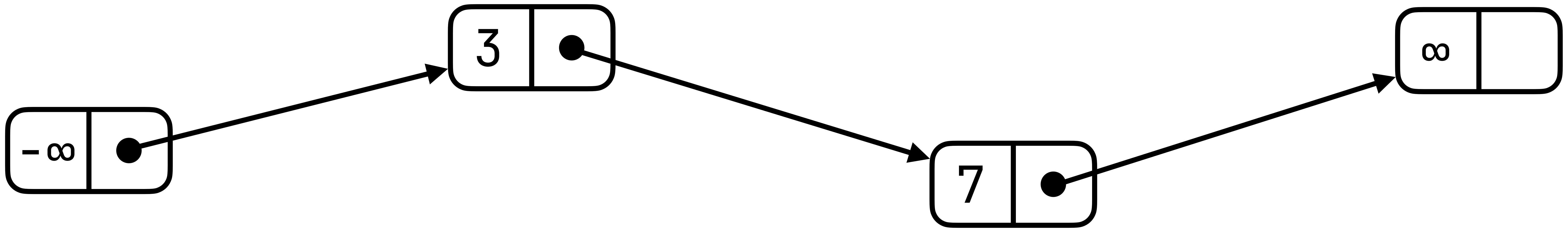

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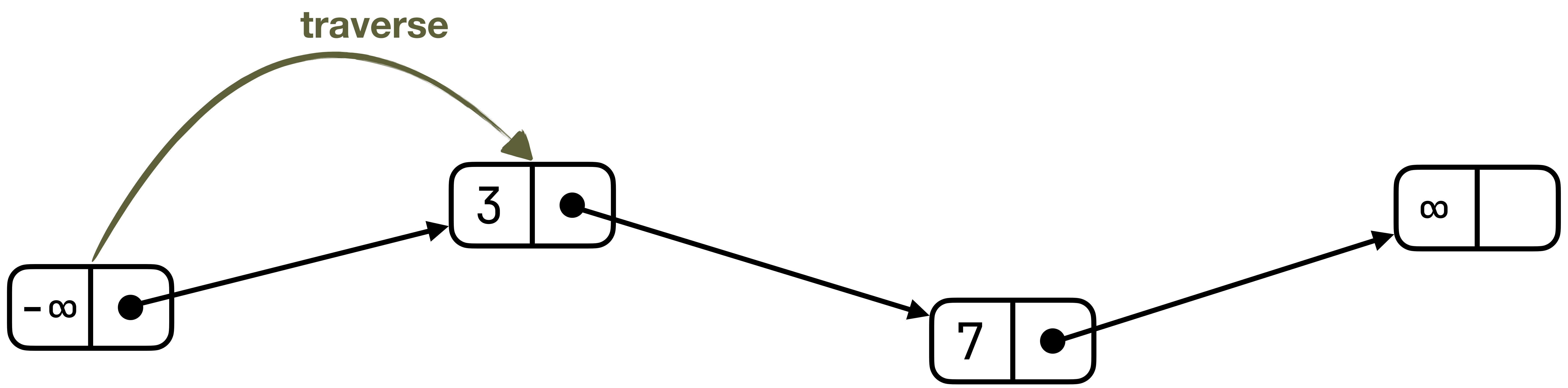

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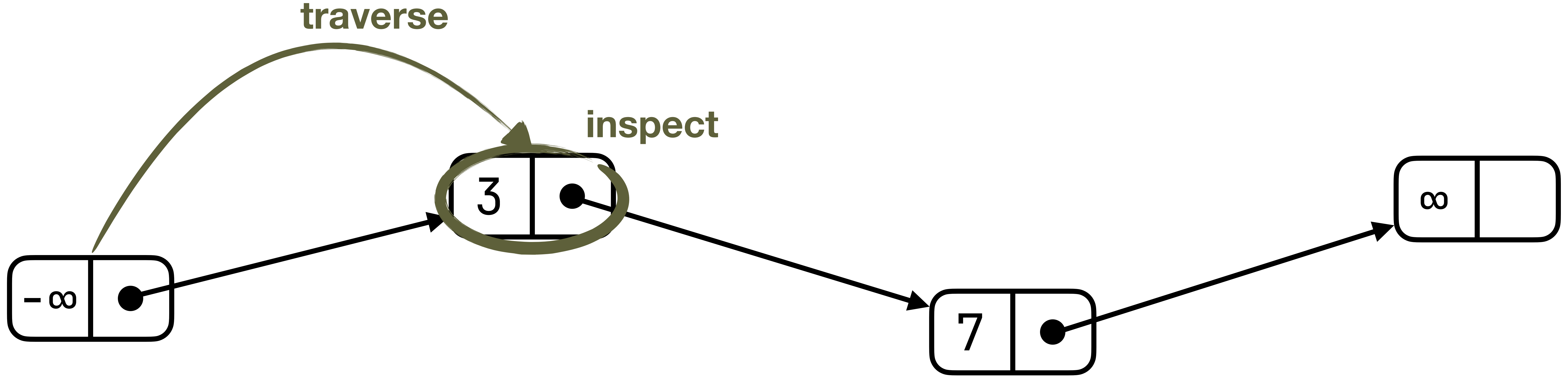

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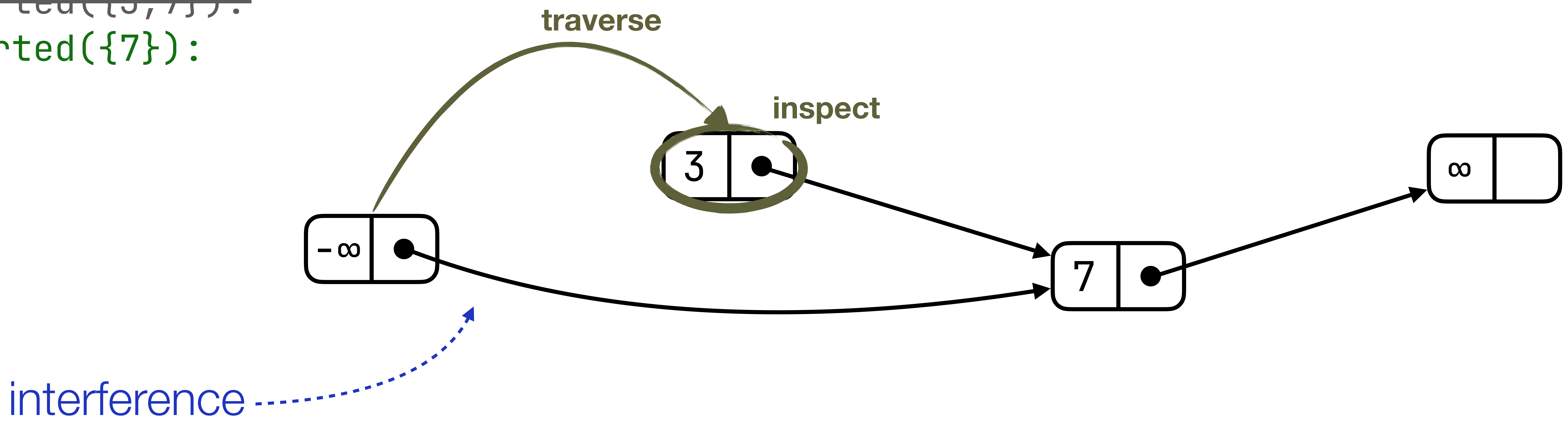

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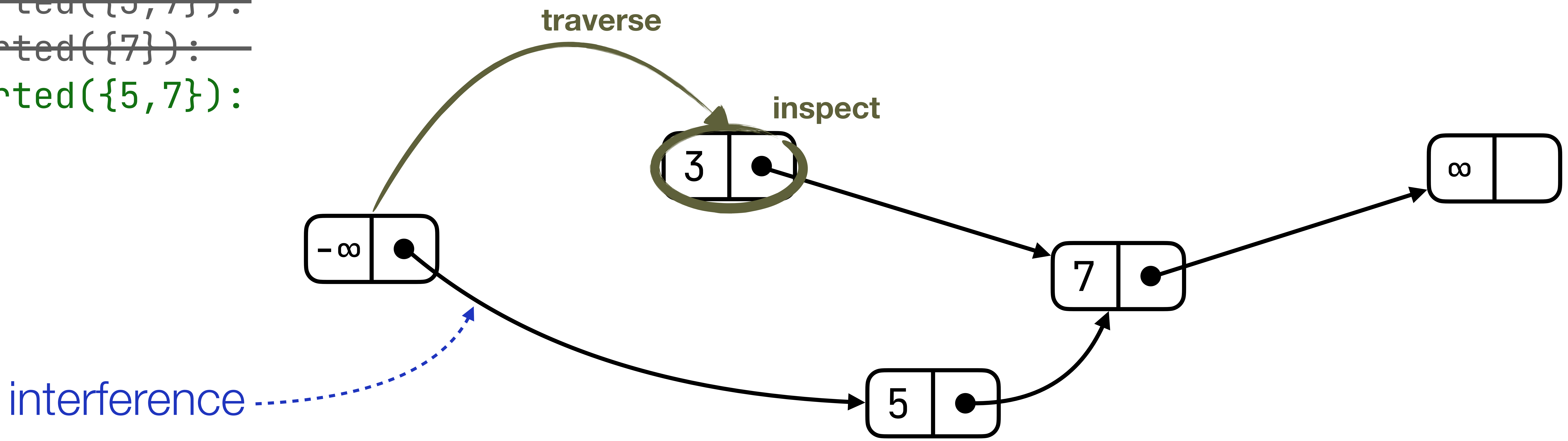

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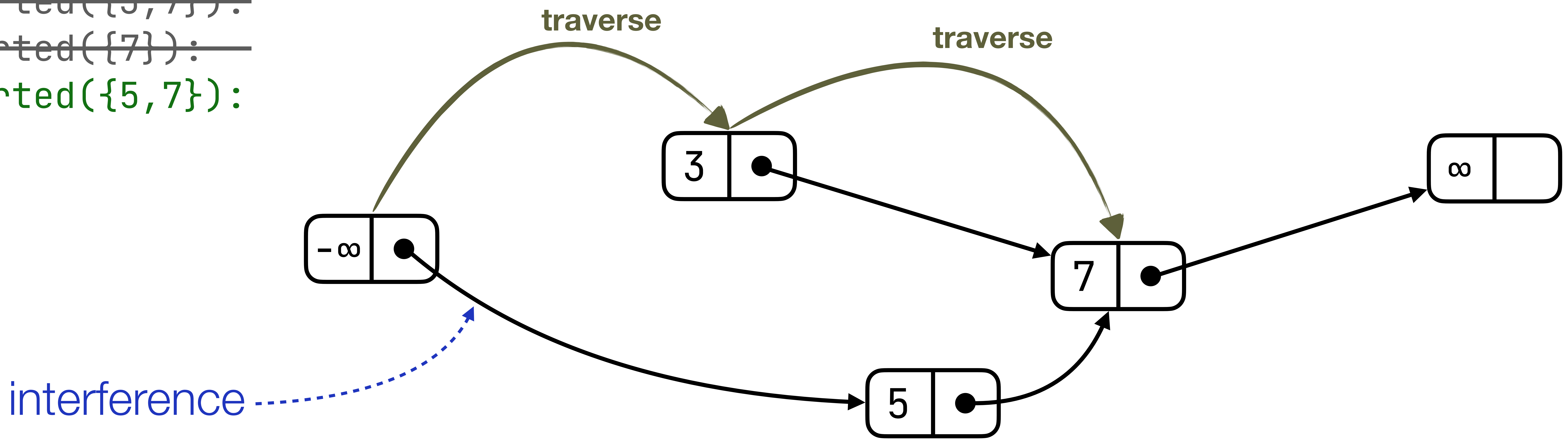

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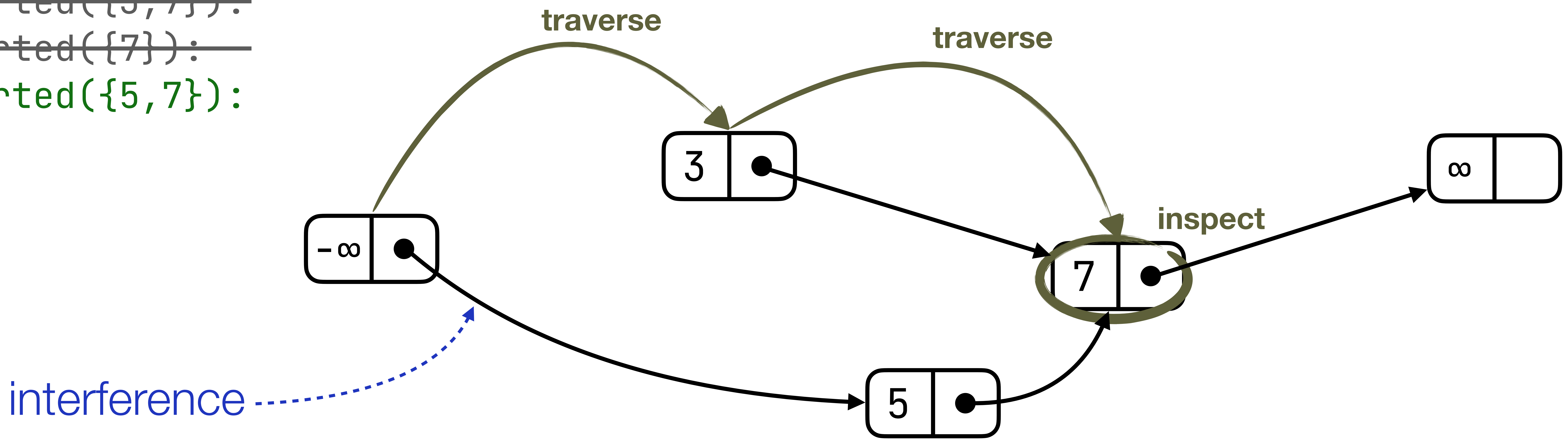

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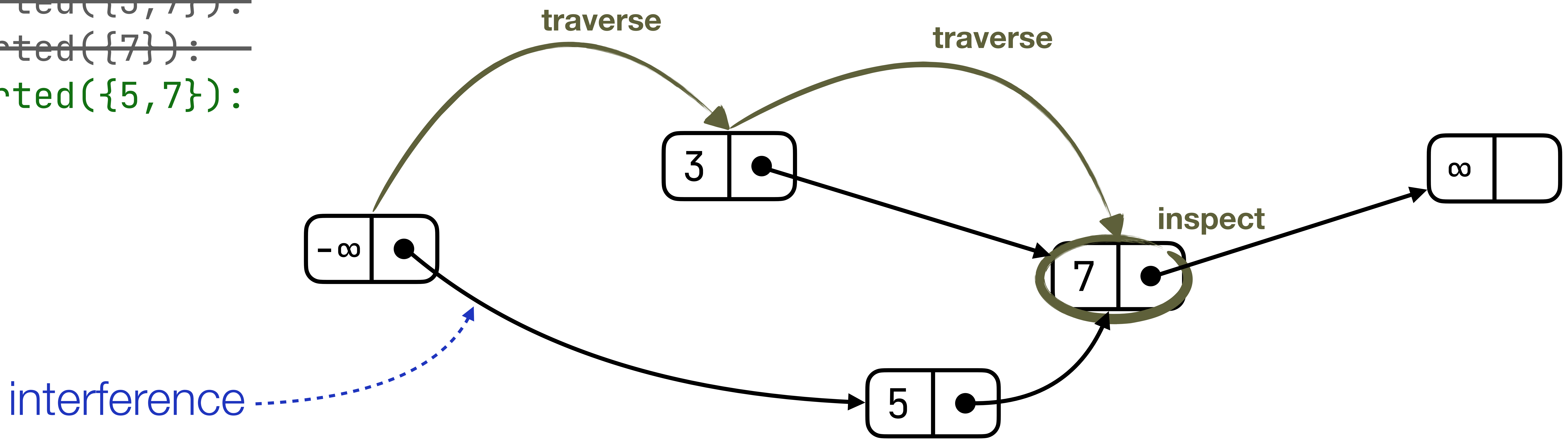

- Result: `contains(5) = false`
  - ➡ linearizable (i.e. correct); proof obligation: `traversal` ⊨ `sorted(M), 5∉M` *`at some point`*

Non-fixed Linearization Points

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Contribution: **hindsight reasoning** in SL

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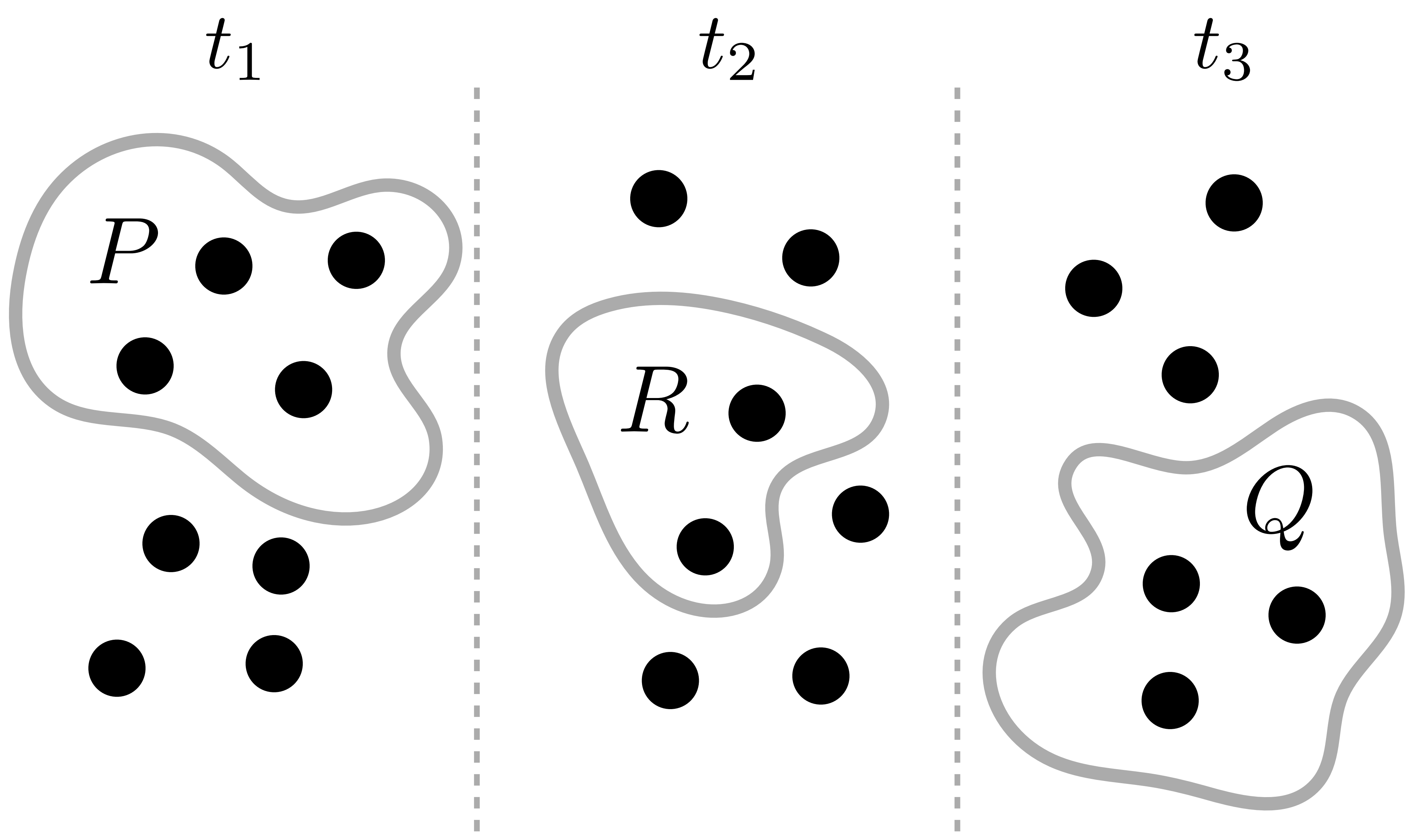

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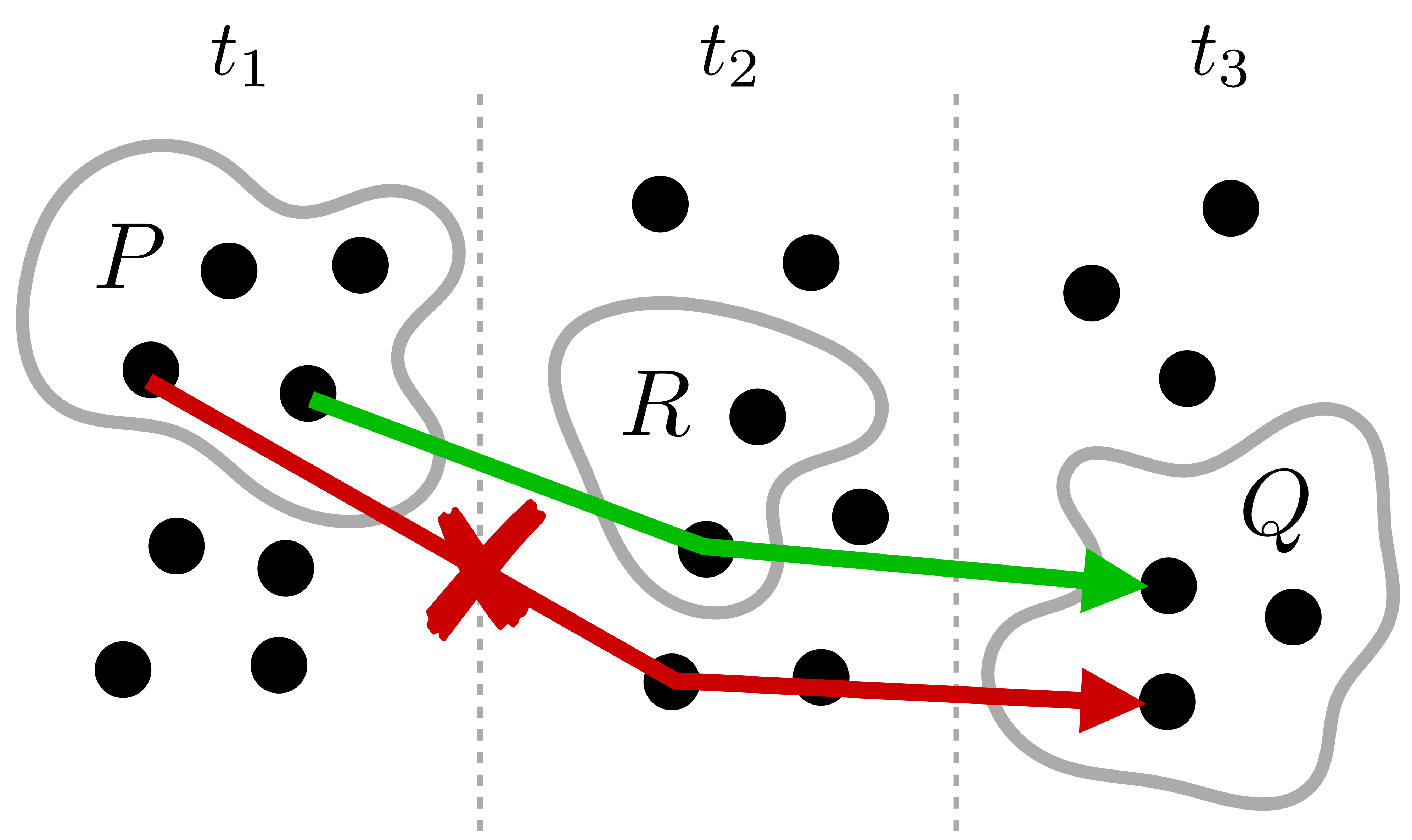

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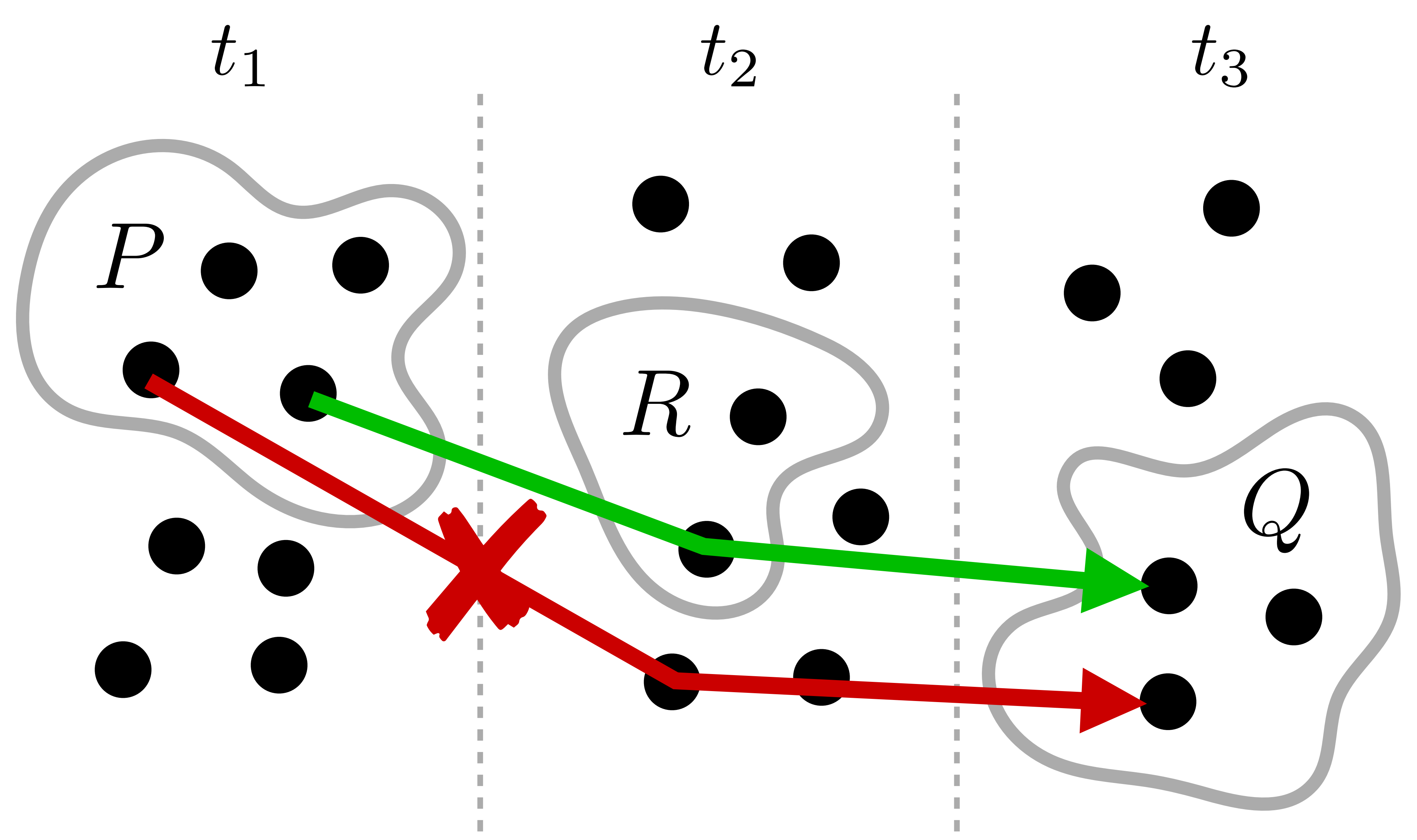

derived from interference recorded by Owicki-Gries / Rely-Guarantee proof

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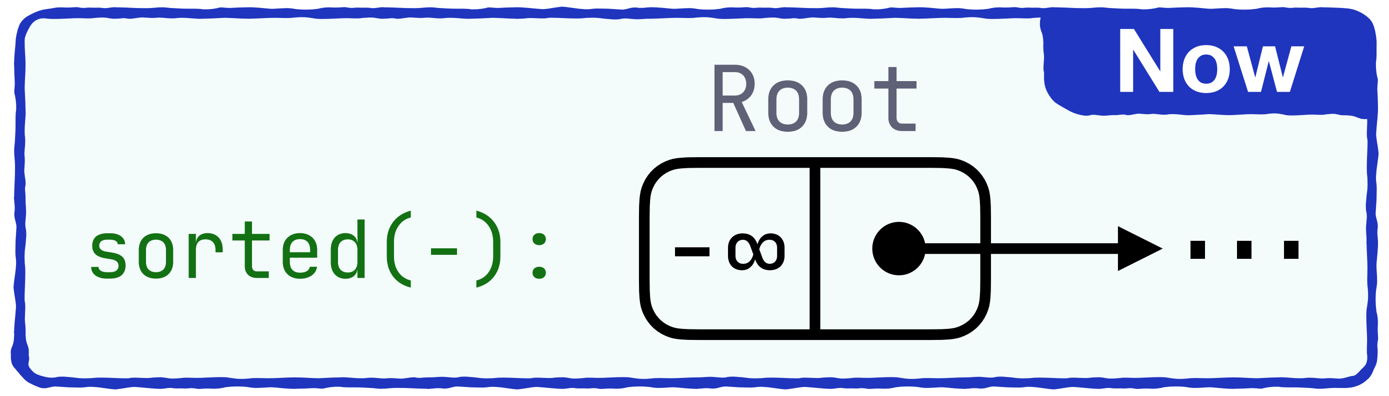

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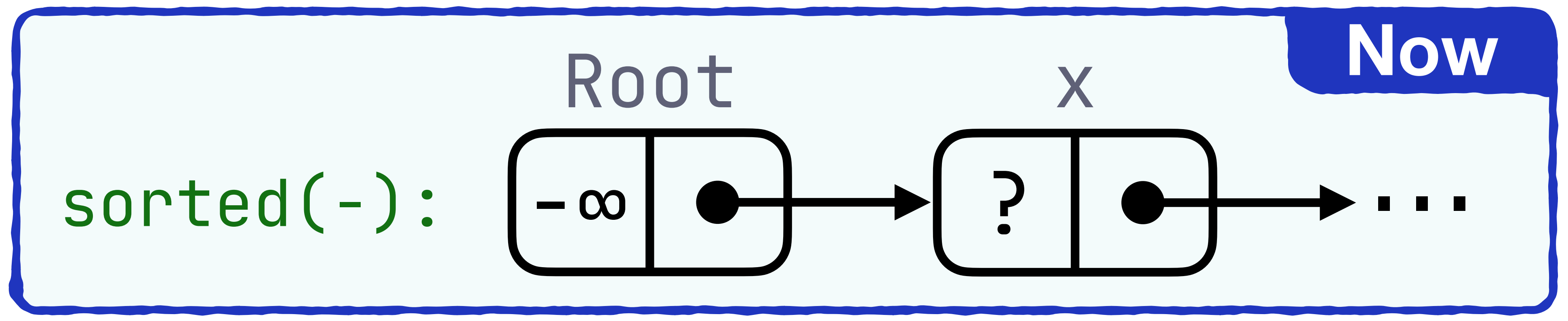

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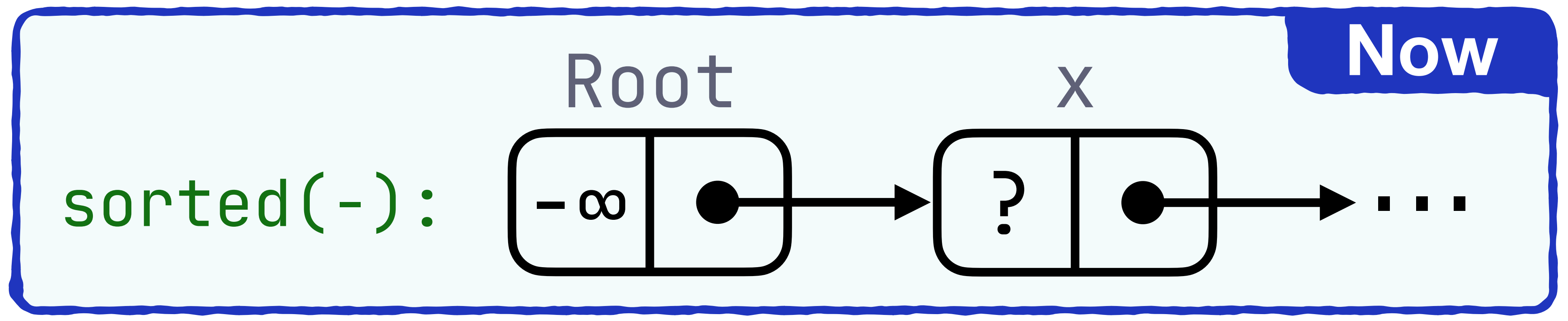

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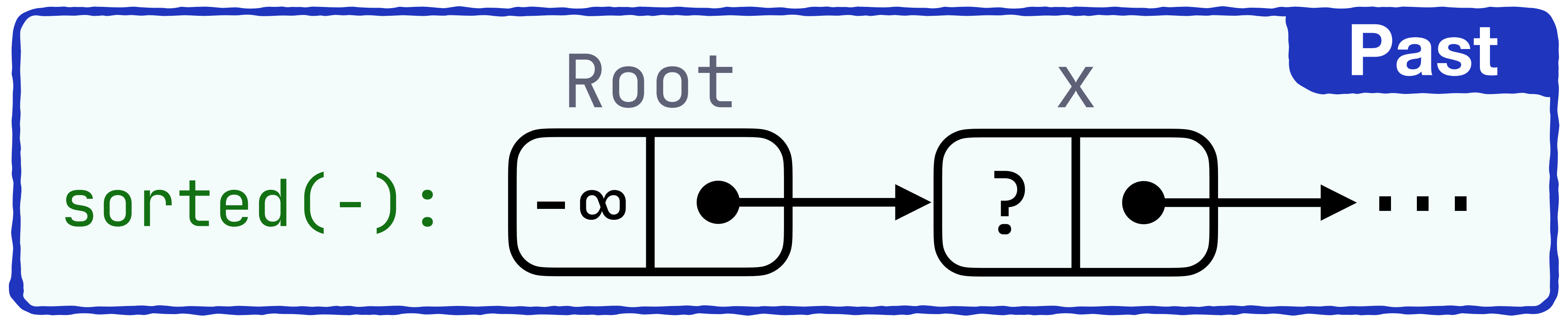

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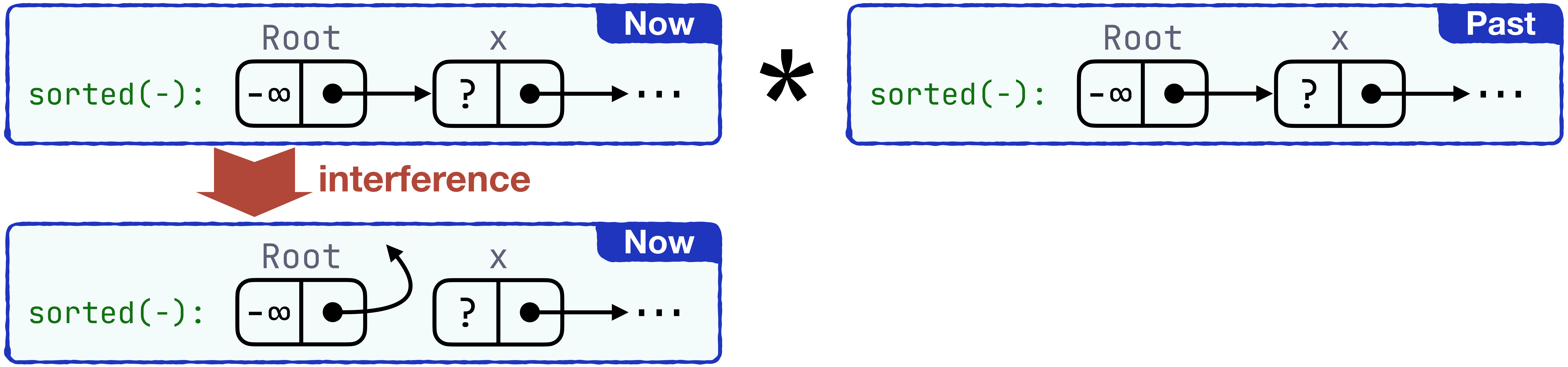

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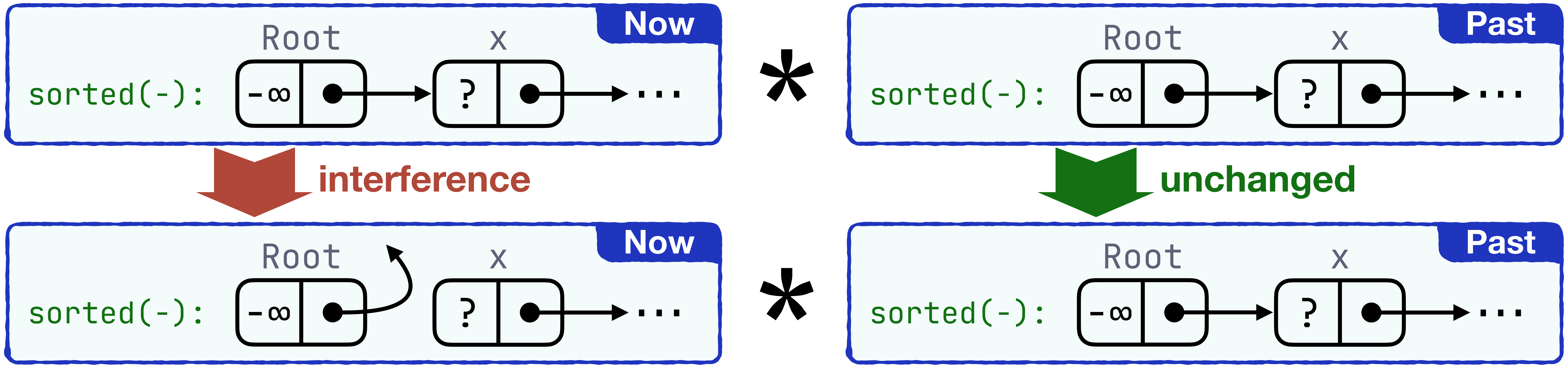

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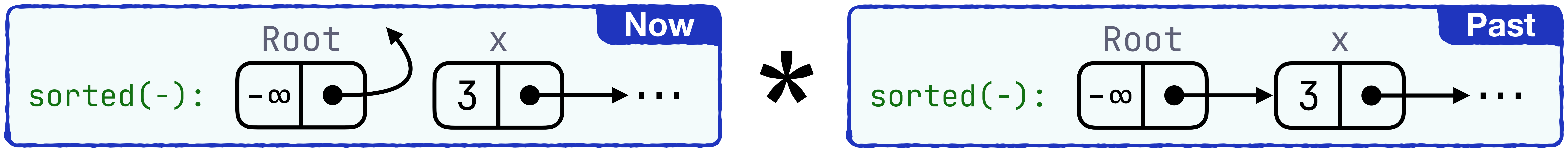

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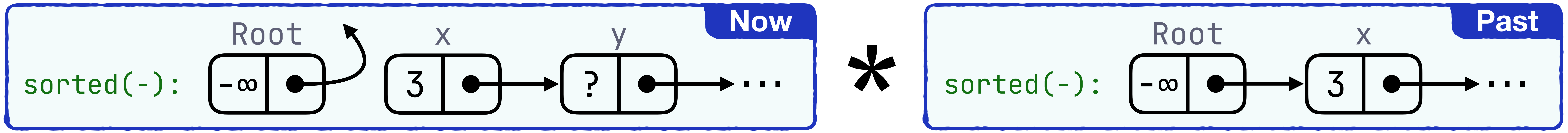

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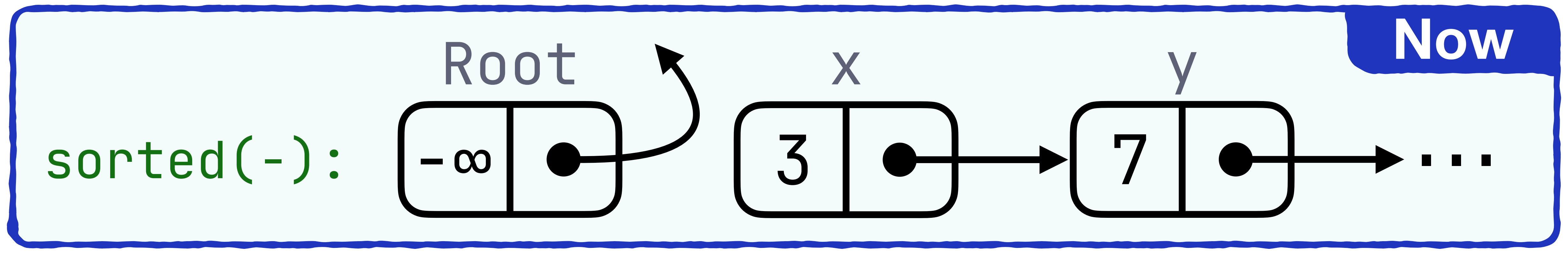

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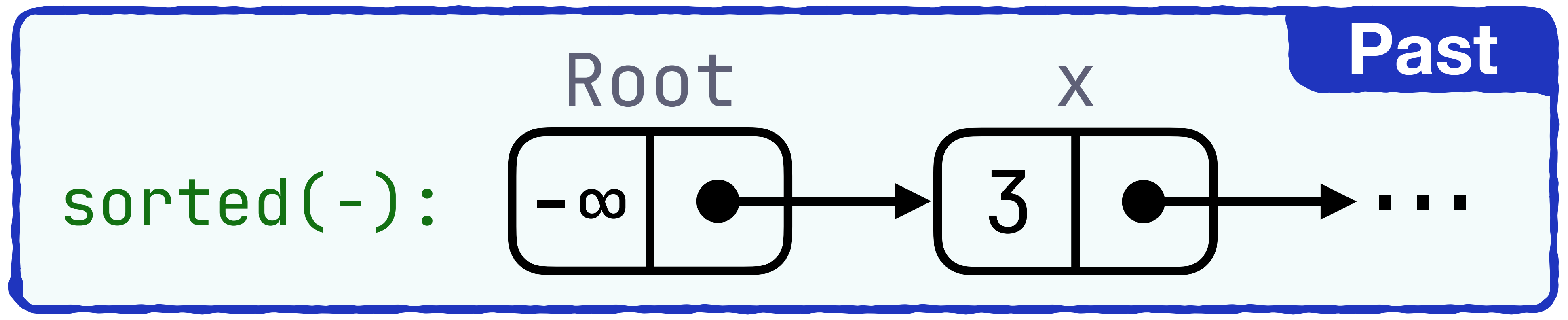

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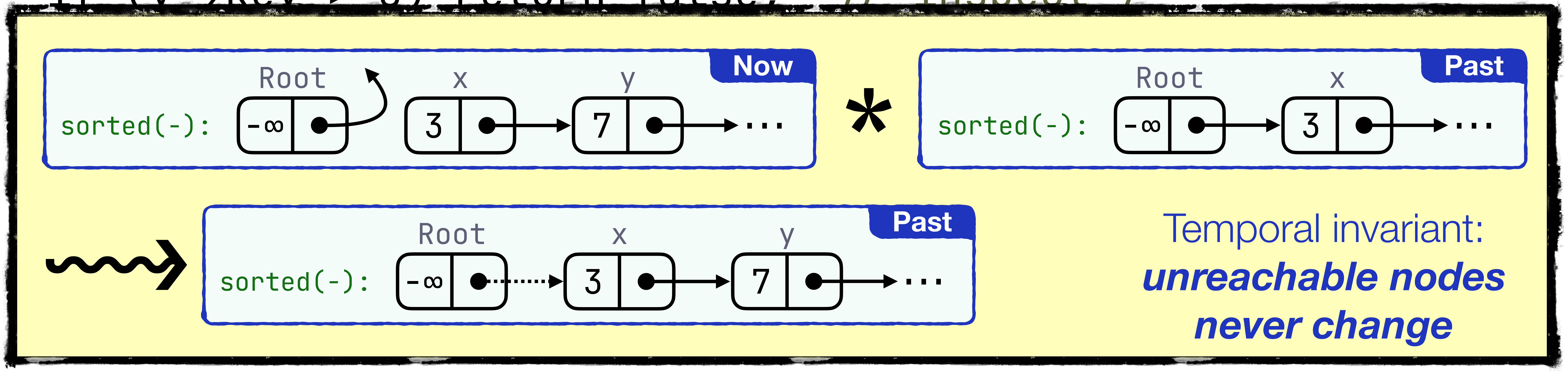

Temporal invariant:
unreachable nodes never change

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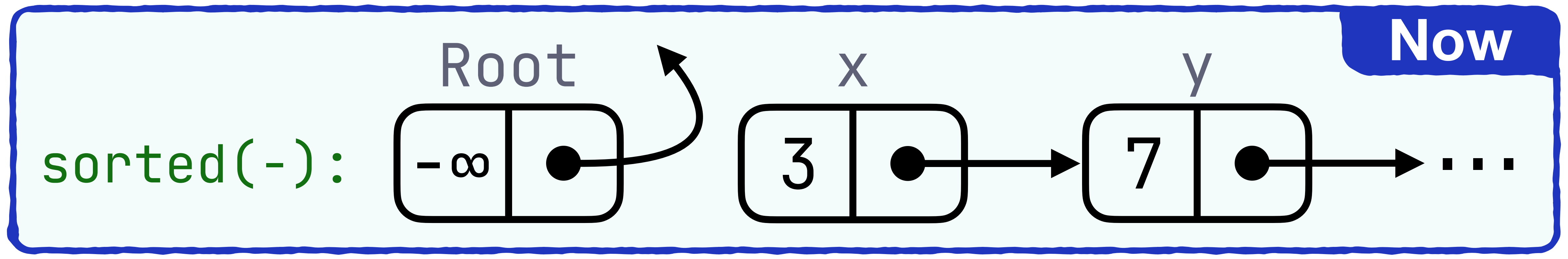

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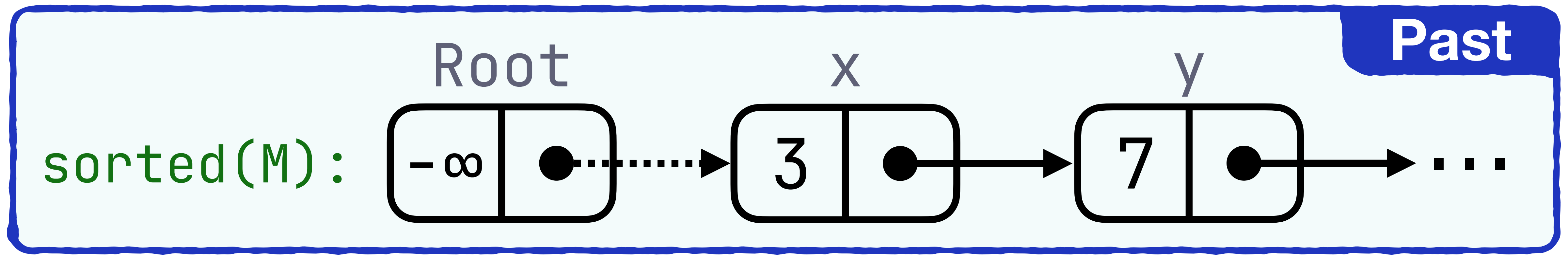

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- Soundness: by elimination of interpolation rule

temporal interpolation
= proof structuring

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- *Logical Ordering Tree*
  - found bug in implementation
  - found bug in previous proof
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Benchmark	Verification Time
Fine-Grained set	45*s* ✓
Lazy set	2*m* 13*s* ✓
FEMRS tree (no maintenance)	3*m* 50*s* ✓
Vechev&Yahav 2CAS set	1*m* 15*s* ✓
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ORVYY set	1*m* 36*s* ✓
Michael set	6*m* 53*s* ✓
Michael set (wait-free search)	6*m* 53*s* ✓
Harris set	57*m* 20*s* ✓
Harris set (wait-free search)	43*m* 00*s* ✓
LO-tree (generalized maintenance)	16*m* 43*s* ✓

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